



- h) If  $u(x, y, z) = 0$  then the value of  $\frac{\partial x}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial x}$  is equal to  
 (A) 1 (B) -1 (C) 0 (D) none of these
- i) The value of  $(1+i)^5 \times (1-i)^5$  is  
 (A) -8 (B)  $8i$  (C) 8 (D) 32
- j) The polar form of the complex number  $\frac{1+i}{1-i}$  is  
 (A)  $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$  (B)  $\sin \frac{\pi}{2} + i \cos \frac{\pi}{2}$  (C)  $\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}$   
 (D)  $\sin \frac{\pi}{4} + i \cos \frac{\pi}{4}$
- k) If  $z = re^{i\theta}$ , then  $|e^{iz}|$  is equal to  
 (A)  $e^{r \sin \theta}$  (B)  $e^{-r \sin \theta}$  (C)  $e^{-r \cos \theta}$  (D)  $e^{r \cos \theta}$
- l) If the system of equations  $x - ky - z = 0$ ,  $kx - y - z = 0$ ,  $x + y - z = 0$  has a non-zero solution, then value of  $k$  can be  
 (A) -3 (B)  $-\frac{1}{2}$  (C) 1 (D) 2
- m) An eigenvalue of a square matrix  $A$  is  $\lambda = 0$ . Then  
 (A)  $|A| \neq 0$  (B)  $A$  is symmetric (C)  $A$  is singular (D)  $A$  is skew-symmetric
- n) If every minor of order  $r$  of a matrix  $A$  is zero, then rank of  $A$  is  
 (A) greater than  $r$  (B) equal to  $r$  (C) less than or equal to  $r$   
 (D) less than  $r$

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions (14)**

a) If  $y = a \cos(\log x) + b \sin(\log x)$  then prove that (5)

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$$

b) If  $y = \frac{1}{x^2 + a^2}$  then find  $y_n$ . (5)

c) Expand  $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$  in powers of  $(x-3)$ . (4)

**Q-3 Attempt all questions (14)**

a) Expand  $f(x) = \sec x$  in powers of  $x$  up to  $x^4$  by Maclaurin's series. (5)

b) Prove that  $\cos^{-1}[\tanh(\log x)] = \pi - 2\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \dots\right)$ . (5)

c) If  $y = \sin^4 x$  then find  $y_n$ . (4)

**Q-4 Attempt all questions (14)**

a) Evaluate:  $\lim_{x \rightarrow 0} \left( \frac{a^x + b^x + c^x + d^x}{4} \right)^{\frac{1}{x}}$  (5)

b) If  $u = f(r)$  and  $r^2 = x^2 + y^2$  then prove that  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$ . (5)



c) If  $z = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$  then show that  $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$ . (4)

**Q-5 Attempt all questions** (14)

a) If  $u = \log(x^3 + y^3 + z^3 - 3xyz)$  then show that  $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$ . (5)

b) Evaluate:  $\lim_{x \rightarrow 1} \left( \frac{x}{x-1} - \frac{1}{\log x} \right)$  (5)

c) Evaluate:  $\lim_{x \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + x^2}{x^3}$  (4)

**Q-6 Attempt all questions** (14)

a) Solve the equation  $z^3 = i(z-1)^3$ . (5)

b) Using De Moivre's theorem prove the following: (5)

$$\cos 5\theta = 5\cos\theta - 20\cos^3\theta + 16\cos^5\theta$$

c) Check whether the following set of vectors is linearly dependent or linearly independent: (4)

$$(1, 2, -1, 0), (1, 3, 1, 2), (4, 2, 1, 0), (6, 1, 0, 1)$$

**Q-7 Attempt all questions** (14)

a) Examine for consistency and if consistent solve them (5)

$$x + 2y = 3, y - z = 2, x + y + z = 1.$$

b) Find the eigen values and eigen vectors of the matrix  $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$ . (5)

c) If  $\sin(\alpha + i\beta) = x + iy$  then prove that  $x^2 \cos^2 \alpha - y^2 \sec^2 \alpha = 1$ . (4)

**Q-8 Attempt all questions** (14)

a) If  $x_r = \cos \frac{\pi}{2^r} + i \sin \frac{\pi}{2^r}$  then prove that  $\lim_{n \rightarrow \infty} x_1 x_2 x_3 \dots x_n = -1$ . (5)

b) Reduce the matrix  $A = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$  to the normal form and find its rank. (5)

c) Discuss the continuity of the function (4)

$$f(x, y) = \frac{x}{\sqrt{x^2 + y^2}} \text{ when } (x, y) \neq (0, 0) \text{ and } f(x, y) = 2 \text{ when } (x, y) = (0, 0).$$

